

BARI-TH/96-233
September 1996

Heavy quark kinetic energy in B mesons by a QCD relativistic potential model

Fulvia De Fazio ¹

Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

Dipartimento di Fisica, Università di Bari, Italy

ABSTRACT

The matrix element of the kinetic energy operator between B meson states is computed by means of a QCD relativistic potential model, with the result: $\mu_\pi^2 = 0.66 \text{ GeV}^2$. A comparison with the outcome of other theoretical approaches and a discussion of the phenomenological implications of this result are carried out.

¹e-mail address: DEFAZIO@BARI.INFN.IT

1 Introduction

In the last two decades, the study of hadronic processes involving heavy quarks has attracted continuous interest both from experimental and theoretical sides. The main theoretical achievements have been obtained in the framework of Heavy Quark Effective Theory (HQET) [1], which describes the dynamics of heavy hadrons, i.e. hadrons containing a heavy quark Q , when $m_Q \rightarrow \infty$. The theory is based upon an effective lagrangian written in terms of effective fields, which is a systematic expansion in the inverse powers of the heavy quark mass m_Q . In particular, it has been pointed out that the expansion in the inverse powers of m_Q is nothing else but an application of Operator Product Expansion (OPE) in the sector of heavy hadrons [2]. The leading order effective Lagrangian displays heavy quark spin and flavour symmetries which are not present in full QCD. These symmetries are no longer conserved at the next-to-leading order, and the $\mathcal{O}(\frac{1}{m_Q})$ lagrangian reads as follows:

$$\mathcal{L} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v [D^2 - (v \cdot D)^2] h_v + \frac{g_s}{2m_Q} \bar{h}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{2} h_v + \mathcal{O}(\frac{1}{m_Q^2}). \quad (1)$$

In the $m_Q \rightarrow \infty$ limit the field h_v is related to the heavy quark field Q by: $Q = e^{-im_Q v \cdot x} h_v(x)$, where v_μ is the heavy quark four-velocity which, in the $m_Q \rightarrow \infty$ limit, coincides with the one of the hadron [3]. In the hadron rest frame, the first of the two next-to-leading order operators appearing in (1) is the heavy quark non relativistic kinetic energy due to its residual motion, while the second one is the Pauli chromomagnetic interaction operator; they correspond in the Wilson expansion to dimension 5 operators. Their matrix elements can be parametrized as follows:

$$\mu_\pi^2(H_Q) = \frac{\langle H_Q | \bar{h}_v (i\vec{D})^2 h_v | H_Q \rangle}{2M_{H_Q}} \quad (2)$$

$$\mu_G^2(H_Q) = \frac{\langle H_Q | \bar{h}_v \frac{g_s}{2} \sigma_{\mu\nu} G^{\mu\nu} h_v | H_Q \rangle}{2M_{H_Q}} \quad (3)$$

where H_Q denotes generically a hadron containing the heavy quark Q and $\mathcal{D}_\mu = \partial_\mu - igA_\mu$ is the covariant derivative. The normalization: $\langle H_Q | H_Q \rangle = 2M_{H_Q}$ is understood.

Since μ_G^2 represents the chromomagnetic interaction between the heavy quark spin \vec{s}_Q and the light cloud total angular momentum \vec{s}_ℓ , it can be obtained from the measured hyperfine mass splitting, when available. Its general expression reads: $\mu_G^2(H_Q) =$

$-2[J(J+1) - \frac{3}{2}]\lambda_2$, where J is the total spin and λ_2 is independent of the heavy quark mass. Therefore, in case of B mesons, we have:

$$\mu_G^2(M_b) = \frac{d_M}{4}(M_{B^*}^2 - M_B^2). \quad (4)$$

where $M_b = B, B^*$ and $d_M = 3$ in the pseudoscalar case, $d_M = -1$ in the vector case (hence, from experimental data [4]: $\mu_G^2(B) \simeq 0.36 \text{ GeV}^2$, $\mu_G^2(B^*) \simeq -0.12 \text{ GeV}^2$). On the other hand, it is expected to be zero for all baryons whose light cloud is in a $\vec{s}_\ell = 0$ state, such as Λ_Q, Ξ_Q , while it should not vanish in the case of Ω_Q for which $\vec{s}_\ell = 1$, though it is not experimentally known, yet. Moreover, the mass splitting has been measured in the case of Σ_b : $M_{\Sigma_b^*} - M_{\Sigma_b} = 56 \pm 16 \text{ MeV}$ [5].

μ_π^2 represents the average square momentum carried by the heavy quark inside the hadron, that is, modulo a factor $2m_Q$, its non relativistic kinetic energy.

These quantities are interesting for several reasons. Heavy hadrons masses are expected to scale with m_Q as:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \dots \quad (5)$$

$\bar{\Lambda}$ represents the difference between the mass of the hadron and that of the heavy quark in the $m_Q \rightarrow \infty$ limit. In this limit, it can be related to the trace anomaly of QCD [6]: $\bar{\Lambda} = \frac{1}{2M_{H_Q}} < H_Q | \frac{\beta(\alpha_s)}{4\alpha_s} G^{\mu\nu} G_{\mu\nu} | H_Q >$, where β is the Gell-Mann-Low function. Moreover, if the inclusive semileptonic width of a heavy hadron is calculated by an expansion in the powers of $\frac{1}{m_Q}$, the following results are found: the leading term of the expansion coincides with the free quark decay rate (spectator model); no corrections of order $\frac{1}{m_Q}$ affect the rate; the $\frac{1}{m_Q^2}$ corrections depend on μ_π^2 and μ_G^2 [7]. The absence of $\frac{1}{m_Q}$ corrections is sometimes referred to as CGG/BUV theorem. As a consequence, these parameters enter in the ratio of hadron lifetimes and in the lepton spectrum in inclusive transitions, which in principle are quantities directly comparable with experimental data ².

It is worth noticing that μ_π^2 and μ_G^2 are the matrix elements of operators which are not sensitive to the light quarks flavour, i.e. they are $SU(3)$ singlet operators. The $SU(3)$ breaking effects emerge at $\frac{1}{m_Q^3}$ level, due to four-quark operators; their matrix elements can be estimated by factorization in the case of mesons, and, in the case of baryons, by constituent quark models [9] or field theoretical approaches, for example QCD sum rules [10].

²A critical analysis of such a procedure can be found in [8].

In this work we calculate μ_π^2 in the case of B mesons by means of a relativistic potential model. In the next section, after describing the relevant features of the model, we will present our results. Phenomenological implications and comparison with other approaches will be provided in section 3. Finally, we will draw our conclusions.

2 Method and Results

We will describe in the following the relativistic quark model used to compute μ_π^2 . Within this model, the state of a pseudoscalar ($b\bar{q}_a$) meson is written in terms of a wave function ψ_B and of quark and antiquark creation operators; in the meson rest frame it reads:

$$|B_a\rangle = i \frac{\delta_{\alpha\beta}}{\sqrt{3}} \frac{\delta_{rs}}{\sqrt{2}} \int d\vec{k} \psi_B(\vec{k}) b^\dagger(\vec{k}, r, \alpha) d_a^\dagger(-\vec{k}, s, \beta) |0\rangle . \quad (6)$$

In (6) α, β are colour indices, r, s are spin indices and a is a light flavour index; the operator b^\dagger creates the b quark with momentum: \vec{k} , while d_a^\dagger creates a \bar{q}_a antiquark with momentum $-\vec{k}$. The wave function ψ_B is obtained as a solution of a Salpeter equation [11] which takes into account relativistic effects in the quark kinematics:

$$\left\{ \sqrt{\vec{k}^2 + m_b^2} + \sqrt{\vec{k}^2 + m_{q_a}^2} - M_B \right\} \psi_B(\vec{k}) + \int d\vec{k}' V(\vec{k}, \vec{k}') \psi_B(\vec{k}') = 0 . \quad (7)$$

Eq. (7) stems from the quark-antiquark Bethe-Salpeter equation in the approximation of instantaneous interaction. The interquark potential V is represented by the Richardson potential [12], which reads in the r -space:

$$V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left[\Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right] ; \quad (8)$$

Λ is a parameter (chosen at the value $\Lambda = 397 \text{ MeV}$ [13]), n_f is the number of active flavours, and the function $f(t)$ is given by:

$$f(t) = \frac{4}{\pi} \int_0^\infty dq \frac{\sin(qt)}{q} \left[\frac{1}{\ln(1 + q^2)} - \frac{1}{q^2} \right] . \quad (9)$$

The potential (8) is linear at large distances in order to assure QCD confinement; at short distances it behaves as $-\frac{\alpha_s(r)}{r}$, with $\alpha_s(r)$ logarithmically decreasing with the distance r to reproduce the asymptotic freedom property of QCD. Spin interaction effects are neglected since in the case of heavy mesons the chromomagnetic coupling is of order m_Q^{-1} . The masses of the constituent quarks are fixed in such a way that the meson spectrum

of the charmonium, the bottomonium and of the heavy-light systems is reproduced: the fitted values are: $m_b = 4.89 \text{ GeV}$, $m_q = m_u = m_d = 0.038 \text{ GeV}$. The mass of the charm quark has also been fixed: $m_c = 1.452 \text{ GeV}$. Finally, the wave function ψ_B is covariantly normalized:

$$\frac{1}{(2\pi)^3} \int d\vec{k} |\psi_B(\vec{k})|^2 = 2M_B. \quad (10)$$

In the B meson rest frame it is useful to define also the reduced wave function $u_B(k)$ ($k = |\vec{k}|$):

$$u_B(k) = \frac{k \psi_B(k)}{\sqrt{2\pi}} \quad (11)$$

which is normalized as: $\int_0^\infty dk |u_B(k)|^2 = 2M_B$. The function $u_B(k)$ can be obtained by numerically solving eq. (7) using the Mulhopp method described in [14, 13]. The B -meson wave function, together with the wave functions computed for the other mesonic states analyzed within this framework, is the main outcome of the model; by using it a number of hadronic quantities characterizing the B system have been computed, such as semileptonic form factors, leptonic decay constants and strong coupling constants [13, 15].

By writing the heavy field, in the expression of the kinetic energy operator, in terms of creation operators, and by exploiting usual anticommutation relation and the normalization condition for the wave function, we obtain a simple expression for the b -quark average momentum squared in the B meson:

$$K(m_b) = \frac{\int_0^\infty dk k^2 |u_B(k)|^2}{\int_0^\infty dk |u_B(k)|^2}. \quad (12)$$

The parameter $\mu_\pi^2(B_d)$ should coincide with $K(m_b)$ in the limit $m_b \rightarrow \infty$. In order to perform such limit, we compute $K(m_b)$ for several values of m_b , using the wave function obtained in correspondence to the appropriate value of m_b ; the result is plotted in Fig. 1. We then extrapolate the resulting curve to $m_b \rightarrow \infty$ according to the expression:

$$K(m_b) = \mu_\pi^2(B_d) \left(1 + \frac{a}{m_b}\right). \quad (13)$$

The result is:

$$\mu_\pi^2(B_d) = 0.66 \text{ GeV}^2 \quad (14)$$

$$a = -1.54 \text{ GeV}. \quad (15)$$

It should be noticed that this extrapolation procedure cannot be avoided in our approach since the resolution of the Salpeter equation can only be performed by numerical methods. Let us observe that the result: $a = -1.54 \text{ GeV}$ suggests that for the charm the $\frac{1}{m_Q}$

corrections play still an important role. On the other hand, the result obtained using the value of m_b fixed within our model, i.e. $m_b = 4.89 \text{ GeV}$, is: $K = 0.46 \text{ GeV}^2$, which means that for the b quark the finite mass result differs from the asymptotic one at the level of 30%. Moreover, eq. (14) agrees quite well with the QCD sum rule result [28].

This result can be translated in a determination of the parameter $\bar{\Lambda}$. As a matter of fact, exploiting eqs. (4) and (5) the following relation can be derived:

$$\bar{M}_B = \frac{3M_{B^*} + M_B}{4} = m_b + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_b} \quad (16)$$

which, using the value in eq.(14), gives:

$$\bar{\Lambda} = 0.35 \text{ GeV} . \quad (17)$$

The results (14) and (17) are also consistent with (5) if one neglects the higher order corrections in $1/m_b$ and uses the experimental value of $\mu_G^2(B_d)$.

Let us briefly discuss the uncertainties of the result in (14). They mainly depend on the computed wave function $u_B(k)$, and, therefore, they can be estimated by modifying the shape of the wave function. We perform this analysis by comparing the outcome in (14) with the result of a similar constituent quark model, such as the one in ref. [16], where the value of $\mu_\pi^2(B_d)$ is obtained by two independent methods. The first one consists in using the Altarelli et al. (ACMM) model [17], where the heavy quark momentum distribution inside the B meson is assumed to be gaussian: $\phi(|k|, P_F) = \frac{4}{\sqrt{\pi}P_F^3} \exp\left(-\frac{|k|^2}{P_F^2}\right)$, P_F being a parameter. In terms of P_F the kinetic energy $\mu_\pi^2(B_d)$ reads: $\mu_\pi^2(B_d) = \frac{3}{2}P_F^2$. The value of P_F has been obtained in [16] by a comparison with recent CLEO data [18] on the inclusive $B \rightarrow X\ell\nu$ semileptonic rate: $P_F = 0.54 \pm_{0.15}^{0.16} \text{ GeV}$, which corresponds to: $\mu_\pi^2 \simeq 0.44 \text{ GeV}^2$. The second method consists in using a quark model, which gives $P_F = 0.5 - 0.6 \text{ GeV}$, i. e. $\mu_\pi^2 = 0.375 - 0.54 \text{ GeV}^2$. We may compare our result with the range of values quoted above: $\mu_\pi^2 = 0.375 - 0.54 \text{ GeV}^2$, observing that the comparison must be performed with our finite mass result, since the wave function used in [16] has been obtained for real values of m_b . By this comparison, we can conservatively conclude that our result is affected by an error of 20% related to the shape of the B meson wave function.

3 Phenomenological implications

Various determinations of the value of $\mu_\pi^2(B_d)$ exist in the literature ³; they are collected in Table I.

The analyses in refs. [16], [20]-[25] consist in various attempts to extract or to put constraints on $\mu_\pi^2(B_d)$ from experimental data. In refs. [20], [23] experimental data on semileptonic B and D decays are compared to theoretical predictions to extract μ_π^2 as a function of $\bar{\Lambda}$; in particular, in ref. [20] the QCD sum rule result [26]: $\bar{\Lambda} = 570 \pm 70 \text{ MeV}$ is used to constrain μ_π^2 in the range: $0.1 < \mu_\pi^2 \leq 1.5 \text{ GeV}^2$. A similar approach is employed in ref. [22], where it is stressed the possibility of obtaining $\bar{\Lambda}$ and μ_π^2 from the moments of the photon spectrum in the decay $B \rightarrow X_s \gamma$.

QCD sum rules have been applied in refs. [27] and [28] to determine μ_π^2 . Moreover, a great deal of works have been devoted to further constrain theoretically μ_π^2 . A field theoretical approach has been applied in ref. [6] (confirmed by a quantum mechanical approach in [29, 30]) to state the inequality: $\mu_\pi^2 > \mu_G^2$ ⁴. Moreover, in [31] a theoretical argument has been given to confirm and strengthen the bound, giving $\mu_\pi^2 > 0.45 \text{ GeV}^2$ in the case of B mesons. This argument is based upon the possibility of extracting μ_π^2 from the slope of the Isgur-Wise function which is related, at the leading $1/m_Q$ order, to the differential decay rate: $\frac{d\Gamma}{dq^2}(B \rightarrow D^* \ell \nu)$ which can be obtained from experimental data [32].

Finally, very recently μ_π^2 has been computed on the lattice [33].

On one hand, this variety of results suggests that further theoretical analyses of μ_π^2 are required, on the other it shows that the experimental determinations are a hard task. The main difficulty lies in the smallness of the parameter μ_π^2 and in the fact that it appears in physically measurable quantities always in connection with quantities that are determined in more or less broad range of values, such as $\bar{\Lambda}$ and the quark masses m_c, m_b .

As an example, we may consider the role played by μ_π^2 in the B semileptonic branching ratio, a well known problem in B physics, since theoretical estimates are still larger than experimental data. The most recent experimental measurement has been performed by CLEO Collaboration [18] giving: $\mathcal{B}(B \rightarrow X \ell \nu) = 10.49 \pm 0.17 \pm 0.43 \%$.

From the point of view of the $\frac{1}{m_Q}$ expansion, the general procedure to determine an inclusive quantity consists in applying the OPE to the forward matrix element of a weak

³ In the baryon sector, a result for the Λ_b has been obtained by QCD sum rules: $\mu_\pi^2(\Lambda_b) \simeq 0.6 \text{ GeV}^2$ [19] with an estimated uncertainty of 30%.

⁴In [24] the bound has been criticized, stating that the role of radiative corrections prevents any possibility of constraining μ_π^2 .

transition operator [2]. The resulting expression for the lepton spectrum in B semileptonic decay, derived in [35], reads:

$$\begin{aligned} \frac{d\Gamma}{dy} &= \Gamma_0 \theta(1 - y - \rho) 2y^2 \left\{ (1 - f)^2 (1 + 2f)(2 - y) + (1 - f)^3 (1 - y) \right. \\ &\quad + (1 - f) \left[(1 - f) \left(2 + \frac{5}{3}y - 2f + \frac{10}{3}fy \right) - \frac{f^2}{\rho} [2y + f(12 - 12y + 5y^2)] \right] \frac{\mu_G^2}{m_b^2} \\ &\quad \left. - \left[\frac{5}{3}(1 - f)^2 (1 + 2f)y + \frac{f^3}{\rho} (1 - f)(10y - 8y^2) + \frac{f^4}{\rho^2} (3 - 4f)(2y^2 - y^3) \right] \frac{\mu_\pi^2}{m_b^2} \right\}, \end{aligned} \quad (18)$$

where:

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{qb}|^2, \quad f = \frac{\rho}{1 - y}, \quad \rho = \frac{m_q^2}{m_b^2}, \quad y = \frac{2E_\ell}{m_b}, \quad (19)$$

and m_q is the mass of the final quark q ⁵.

Let us consider the case $B \rightarrow X_c \ell \nu$. Eq. (18) depends on the value of the charm quark mass m_c , which is fixed in the potential model to the value $m_c = 1.452 \text{ GeV}$ by fitting the whole charmonium spectrum. Using $V_{cb} = 0.04$ and our result (14) we obtain the lepton spectrum displayed in fig. 2. This curve must be compared to the experimental distribution in [18]. Since on the theoretical side, heavy quark mass expansion is unreliable for large lepton energies $E_\ell \geq 2 \text{ GeV}$ [35], and, in the experimental analyses, high energy leptons must be selected in order to subtract the background of secondaries, the comparison theory-experiment can be performed only in a selected window of lepton energies. For example, as it has been done in refs. [24], [25], one can use the ratios:

$$R_1 = \frac{\int_{E_\ell \geq 1.5 \text{ GeV}} E_\ell \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_\ell \geq 1.5 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell}, \quad R_2 = \frac{\int_{E_\ell \geq 1.7 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_\ell \geq 1.5 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell} \quad (20)$$

where the dependence on the overall factor $|V_{cb}|^2 m_b^5$ cancels. Using the experimental results: $R_1 = 1.7830$ $R_2 = 0.6108$ for the ratios in (20) the values of μ_π^2 displayed in Table I have been obtained [24], [25]⁶. Using the formulae for $R_{1,2}$ in [24] and our results (14), (17), we obtain: $R_1 = 1.733$ $R_2 = 0.559$. However, as already pointed out in [25], the parameters $\bar{\Lambda}$, μ_π^2 enter in $R_{1,2}$ as power corrections and represent a small effect in (20), so that very small changes in the theoretical or in the experimental expressions for $R_{1,2}$ would shift the values of $\bar{\Lambda}$, μ_π^2 towards very different results. One of such changes could be related to a different estimate of the secondary electron background, or, from the

⁵The expression of $\mathcal{O}(\alpha_s)$ corrections to eq. (18) can be found in [36].

⁶The formulae in [24] for $R_{1,2}$ include also $\mathcal{O}(\alpha_s)$ corrections.

theoretical side, to the next order perturbative corrections, whose size is difficult to assess. Therefore, one cannot avoid to conclude that the accuracy of a single determination from experimental data is difficult to check. A set of independent measurements, from different channels, should be used, and an accurate cross check of the errors should be performed to detect the value of the parameter μ_π^2 .

Acknowledgments

I thank P. Colangelo and G. Nardulli for interesting discussions.

References

- [1] For a recent review see for example: B.Grinstein, *An Introduction to Heavy Mesons*, published in: Mexican School 1994, pag. 122 and references therein.
- [2] For a comprehensive review on this subject see for example: M.A. Shifman, *Lectures on Heavy Quarks in Quantum chromodynamics*, preprint TPI-MINN-95/31-T, UMN-TH-1413-95, hep-ph/9510377.
- [3] H. Georgi, Phys. Lett. **B240** 447 (1990).
- [4] Review of particle properties, Phys. Rev. **D50** 1173 (1994).
- [5] P. Abreu et al., DELPHI Collab., preprint DELPHI 95-107 PHYS 542 (1995).
- [6] I. Bigi, M. A. Shifman, N. Uraltsev and A. Vainshtein, Phys. Rev. **D52** 196 (1995).
- [7] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. **B247** 399 (1990); I. Bigi, N. Uraltsev and A. Vainshtein, Phys. Lett. **B293** 430 (1992); (E) **B297** 477 (1993).
- [8] G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, preprint CERN-TH/96-77 , hep-ph/9604202.
- [9] N.Bilić, B.Guberina and J.Trampetic, Nucl. Phys. **B248** 261 (1984).
- [10] P. Colangelo and F. De Fazio, preprint BARI-TH/96-230, hep-ph/9604425.
- [11] E. E. Salpeter, Phys. Rev. **87** 328 (1952).
- [12] J. L. Richardson, Phys. Lett. **B82** 272 (1979).
- [13] P. Colangelo, G. Nardulli and M. Pietroni, Phys. Rev. **D43** 3002 (1991).
- [14] K. Karamcheti, Principles of ideal fluid aerodynamics (Wiley, New York, 1966).
- [15] P. Cea, P. Colangelo, L. Cosmai and G. Nardulli, Phys. Lett. **B206** 691 (1988).
- [16] D. S. Hwang, C. S. Kim and W. Namgung, preprint KEK-TH-473, hep-ph/ 9604225.
- [17] G. Altarelli, N. Cabibbo, G. Corbó, L. Maiani and G. Martinelli, Nucl. Phys. **B208** 365 (1982).
- [18] B. Barish et al., CLEO Collab., Phys. Rev. Lett. **76** 1570 (1996).

- [19] P. Colangelo, C. A. Dominguez, G. Nardulli and N. Paver, Phys. Rev. **D54** 4622 (1996).
- [20] M. Luke and M. J. Savage, Phys. Lett. **B321** 88 (1994).
- [21] A. F. Falk, M. Luke and M. J. Savage, Phys. Rev. **D53** 6316 (1996).
- [22] A. Kapustin and Z. Ligeti, Phys. Lett. **B355** 318 (1995).
- [23] Z. Ligeti and Y. Nir, Phys. Rev. **D49** 4331 (1994).
- [24] M. Gremm, A. Kapustin, Z. Ligeti and M. B. Wise, Phys. Rev. Lett. **77** 20 (1996).
- [25] V. Chernyak, Nucl. Phys. **B457** 96 (1995); preprint Budker INP-96-23, hep-ph/9604376.
- [26] M. Neubert, Phys. Rev. **D46** 1076 (1992).
- [27] V. Eletsky and E. Shuryak, Phys. Lett. **B276** 191 (1992).
- [28] P. Ball and V. M. Braun, Phys. Rev. **D49** 2472 (1994).
- [29] I. Bigi, M. A. Shifman, N. Uraltsev and A. Vainshtein, Int. J. Mod. Phys. **A9** 2467 (1994).
- [30] M. Voloshin, preprint TPI-MINN-94/18-T.
- [31] I. Bigi, A. G. Grozin, M. A. Shifman, N. Uraltsev and A. Vainshtein, Phys. Lett. **B339** 160 (1994).
- [32] B. Barish et al., CLEO Collab., Phys. Rev. **D51** 1014 (1995).
- [33] V. Gimenez, G. Martinelli and C. T. Sachrajda, preprint CERN-TH/96-175, hep-lat/9607055.
- [34] E. Bagan, P. Ball, V. M. Braun e P. Gosdzinsky, Phys. Lett. **B342** 362 (1995) [E: ibid. **B374** 363 (1996)]; E. Bagan, P. Ball, B. Fiol e P. Gosdzinsky, Phys. Lett. **B351** 546 (1995).
- [35] I. I. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Phys. Rev. Lett. **71** 496 (1993).
- [36] M. Jezabek and J. H. Kuhn, Nucl. Phys. **B320** 20 (1989); A. Czarnecki and M. Jezabek, Nucl. Phys. **B427** 3 (1994).

	[21]	[24]	[25]	[16]	[27]	[28]	[33]	This paper
$\mu_\pi^2(B_d)$	0.1	0.19 ± 0.1	0.135	0.44 ± 0.25	0.23 ± 0.11	0.60 ± 0.10	-0.09 ± 0.14	0.66

Table I.

Results for the B_d meson matrix element of the b quark kinetic energy operator.

Figure Captions

Figure 1

The b -quark average square momentum K as a function of $\frac{1}{m_b}$. The straight line results from a two parameters fit.

Figure 2

The lepton spectrum $\frac{d\Gamma}{dE_\ell}$ in the semileptonic decay $B \rightarrow X_c \ell \nu$.

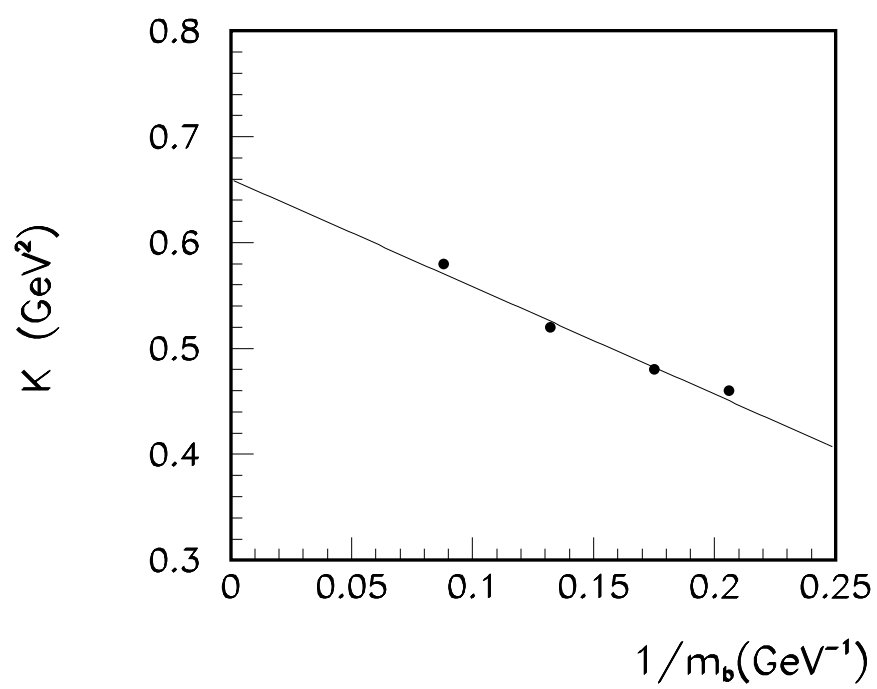


Fig. 1

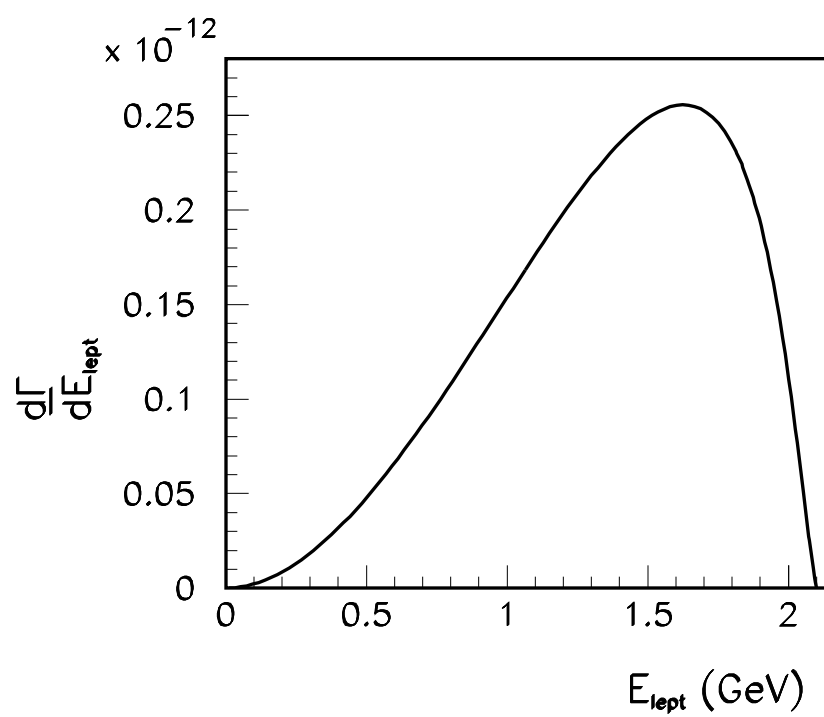


Fig. 2